THE USE OF THE ANALYTIC HIERARCHY PROCESS IN CONFLICT ANALYSIS AND AN EXTENSION

Iwan Azis and Walter Isard
University of Indonesia and Cornell University

I. Introduction

This research note is written to serve as an appendix to the article by Iwan Azis, "New Conflicts Between Developed and Developing Countries," this Journal, Vol. 3, No. 1, 1996. Since knowledge of the *Analytic Hierarchic Process* (AHP) is not widespread among economists and other social scientists, we judge it to be desirable to present its elements and steps for application in cases of conflict that have a game theory flavor.

Also, in writing this note we have been able to extend this process so that the fairly precise relative evaluations of elements derived for each party can be useful for a mediator or third person in resolving a conflict.

AHP is designed to exploit certain information in situations where participants cannot assign a definite value such as utility or dollar's worth to objects or other elements, but where they can do more than just rank the desirability of those objects or elements. The process assumes that they can make pairwise comparisons of elements and state that one element is X times as desirable as a second one--- e.g. government operation of a steel mill is only one-half as desirable as private operation, or that the objective of eliminating unemployment is three times as desirable as that of improving environmental standards.

The steps in this process that can be followed will be set down in two ways: (1) a non-mathematical approach for those wishing to replicate the computation themselves and who have only elementary background in mathematics and (2) a mathematical form that embodies the integrated and more formal representation of the underlying behavior of the participants.

II. Non-Mathematical Statement of the Steps

1. Set down a scale for relative importance for use in making pairwise comparisons. For example, it might be the scale in Table A that Saaty (1976) has often suggested. This is a subjective step. The scale should be one that the analyst finds relevant for the conflict situation, and especially the perspectives and knowledge base of participants in the conflict.

Table A: The Saaty Scale and Its Description.

Intensity of Importance	*Definition*	*Explanation*
1[a]	Equal importance	Two policies contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one policy over another
7	Demonstrated importance	A policy is strongly favored and its dominance is demonstrated in practice
9	Absolute importance	The evidence favoring one policy over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed
Reciprocals of above nonzero numbers	If policy i has one of the above nonzero numbers assigned to it when compared with policy j, then j has the reciprocal value when compared with i	
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix

[a]On occasion in 2 by 2 problems, Saaty has used 1 + *, 0 < * < 1/2 to indicate very slight dominance between two nearly equal activities.

Source: adapted from Saaty and Khouja (1976:34).

2. For each participant, set up the hierarchy of elements consisting of the overall goal, the objectives or sub-goals that are required to attain the overall goal, the targets or instruments required to achieve each objective, and the joint-actions that can be taken to reach these targets, and so forth (see the hierarchy for the developed countries, DCs, in Diagram 1, which is reproduced from Azis, 1996)

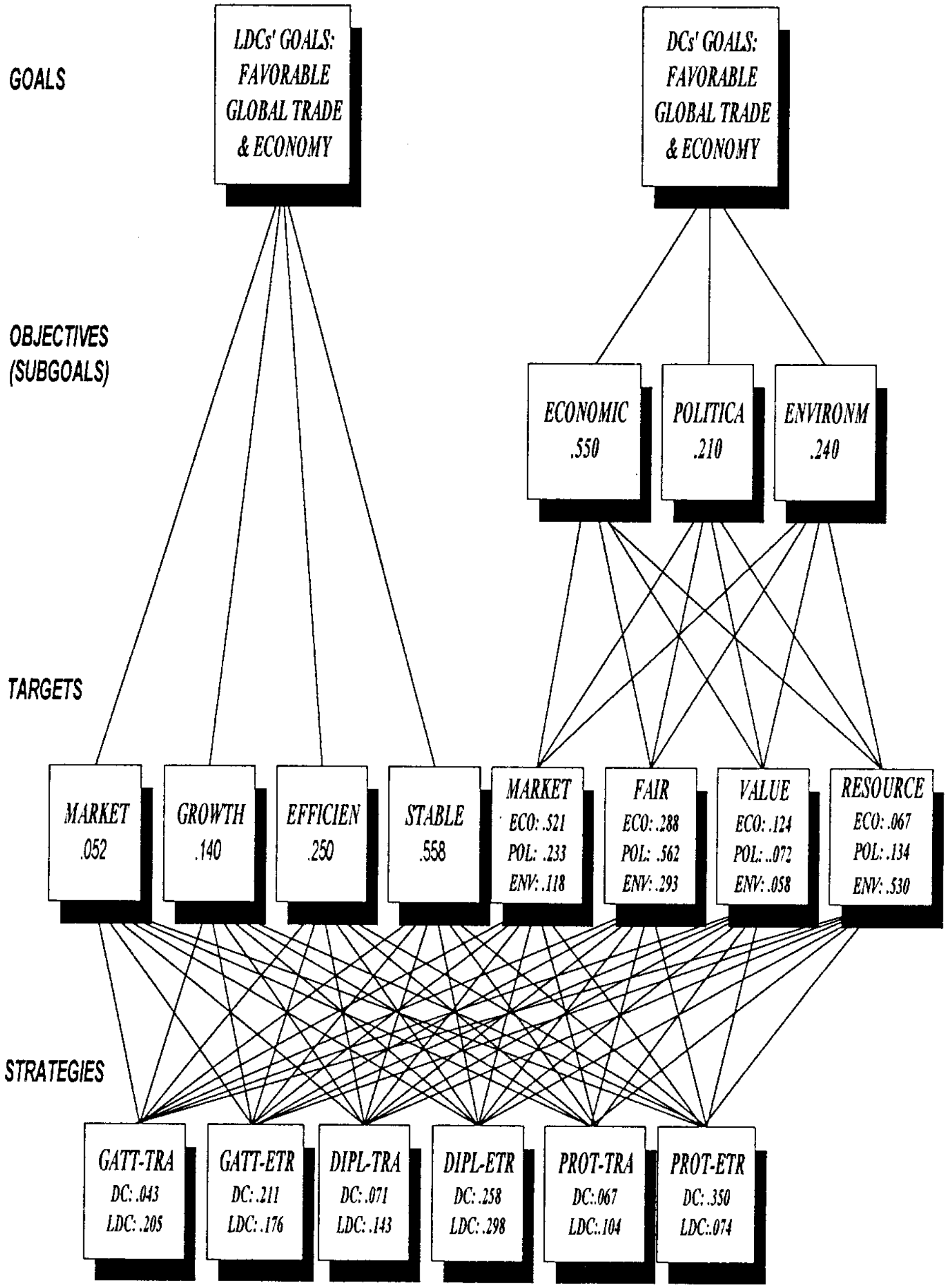

Diagram 1: Non Equilibrium

3. Consider the elements in the second level of a hierarchy, where the top level, a single node, represents the overall goal and the second level corresponds to the targets to achieve that goal, as is the case for the less-developed countries (LDCs). Suppose there are three targets which we may designate A_1, A_2, and A_3. In general, when there are n targets, designate them A_1, A_2, A_3, ...A_n.
4. Set up a table for pairwise comparisons where the above elements are listed in order at the top of the columns and at the left side of rows (as in Table B when there are three elements involved). In general, this table may be designated an n-by-n matrix.
5. For each cell of the matrix relevant for a participant, have the participant state the relevant importance of the row element A_i (i = 1,2,…,n) when compared with the column element A_j (j = 1,2,…n) for achieving the overall goal at the top level. Suppose in a 3-by-3 case, he/she states the relative values noted in the cells of Table B.

Table B: Relative Importance of the Targets to Achieve the Overall Goal

Goal	A1	A2	A3
A1	1	4	5
A2	1/4	1	1
A3	1/5	1	1

The number in the first row is unity, signifying that element A_1 of the row is of equal importance to element A_1 of the column—that is of equal importance to itself. The second number in the first row is 4, signifying that in the mind of the participant A_1 is four times as important as A_2 in achieving the overall goal. The third number in the first row is 5 indicating that A_1 is five times as important as A_3. The second row states that in the mind of the participant, A_2 is one-quarter as important as A_1, that A_2 is equally important to itself; and that A_2 is equally important as A_3. The third row states that A_3 is one-fifth as important as A_1 and equally important as A_2 and itself (A_3).

Clearly the analyst can point out the inconsistencies in the relative values the participant stated. For example, if A_1 is four times as important as A_2 and five times as important as A_3, then A_2 cannot be equally important as A_3 as he/she stated in the pairwise comparisons recorded in the second row. The analyst can then point out that if the relative values in the first row are correct, then to be consistent the values in the second row should be 1/4, 1 and 5/4; and that the values in the third row should be 1/5, 4/5 and 1. The participant may then be urged to restate his/her relative values in a consistent manner.

However, in certain situations, he/she may resist doing so; and clearly when there are more than three or four levels in a table, and a number of elements in each, the participant may find it exceedingly difficult or taxing to do so, and may refuse. In either case, we can proceed.

Whether or not the table depicts consistent numbers, from the set of relative values in a table, obtain a set of normalized weights for the three targets, A_1, A_2, and A_3. First, sum the relative values in each column, yielding the totals of 1.45, 6 and 7 for the first, second and third columns, respectively. Next divide each cell by the total of its column. This yields Table C. Add the cells in

Table C: Weights (Normalized Values) for Each Target

Goal	A1	A2	A3	Totals	Weights
A1	0.69	0.667	0.714	2.071	0.69
A2	0.173	0.167	0.143	0.483	0.161
A3	0.137	0.167	0.143	0.447	0.149

each row and divide by 3, the number of targets. This yields the weights 0.690, 0.161, and 0.149 for the first, second and third targets, respectively.

In the case of the LDCs' hierarchy in Azis (1996), reproduced in Diagram 1, the second level contains four elements, <u>MARKET</u> (access to DCs' markets), <u>GROWTH</u> (strong economic growth), <u>EFFICIEN</u> (greater efficiency) and <u>STABLE</u> (political stability). These four are listed by rows and columns of Table D. From interviews with several LDCs' scholars well informed on trade issues, relative values in Table D were derived from pairwise comparisons of the relative importance of these targets for achieving the LDCs' overall goal (favorable global trade and economy). It is clear that the values in Table D are inconsistent.

Table D: Ranking of Targets With Respect to LDCs' Goals

LDCs' goal	MARKET	GROWTH	EFFICIEN	STABLE
MARKET	1	1/5	1/4	1/7
GROWTH	5	1	1/3	1/5
EFFICIEN	4	3	1	1/3
STABLE	7	5	3	1
Total	17	9.2	4.58	1.676

When the normalization procedure outlined in the previous paragraphs is followed, we obtain the weights of 0.052, 0.149, 0.245, and 0.552 for the four targets. These weights are noted under the respective targets at the left of each row in Table F.

6. If the hierarchy possesses a third level, as is the case for the LDCs, then the next step is to determine the normalized weights of the set of elements (here the six joint-actions) listed in the third level for each of the elements (i.e., for each of the targets) listed in the second level. For example, from interviews with the well-informed LDCs' scholars, judgments via pairwise comparisons were obtained on the relative importance of the six joint-actions (listed as rows and columns in Table E) for the attainment of the MARKET target. They are recorded in the cells of Table E. Following the procedure outlined above, the normalized weights of the six joint-actions were obtained; they are recorded in the first row of Table F. We do the same for determining the normalized weights of the six joint-actions for the attainment of the LDCs' second target, GROWTH, and record the normalized weights in the second row of Table F. In the same manner, we obtain the normalized weights for the EFFICIEN and STABLE targets, recorded respectively in the third and fourth rows of Table F.

Table E: Relative Importance of the Six Joint Actions for the Attainment of the MARKET Target

GROWTH	GATT-TRA	GATT-ETR	DIPL-TRA	DIPL-ETR	PROT-TRA	PROT-ETR
GATT-TR	1	1/4	1/2	1/4	5	2
GATT-ET	4	1	3	1/4	5	4
DIPL-TRA	2	1/3	1	1/5	4	1/2
DIPL-ETR	4	4	5	1	7	6
PROT-TR	1/5	1/5	1/4	1/7	1	1/3
PROT-ET	1/2	1/4	2	1/6	3	1
Total	11.7	6.033	11.75	2.009	25	13.833

Table F: Relative Values of Joint Actions for Gaining Each LDCs' Target

	GATT-TRA	GATT-ETR	DIPL-TRA	DIPL-ETR	PROT-TRA	PROT-ETR
MARKET (0.052)	0.1064	0.2294	0.1011	0.4403	0.0345	0.0883
GROWTH (0.149)	0.4647	0.1079	0.1884	0.0514	0.1509	0.0366
EFFICIEN (0.245)	0.3957	0.0934	0.1984	0.0864	0.1984	0.0277
STABLE (0.552)	0.0595	0.2231	0.1182	0.4338	0.0604	0.1052

7. To obtain the relative importance of each of the six joint-actions for attaining the overall goal when each has an effect through its contribution to the MARKET target, we multiply the first row in Table F by 0.052, the normalized weight of the MARKET target for attaining the overall goal. We record the resulting six products in the first row of Table G. To obtain the relative importance of each of the six joint-actions for the attainment of the overall goal when each has an effect through its contribution to the GROWTH target, we multiply the second row in Table F by 0.149, the normalized weight of the GROWTH target; the results are recorded in the second row of Table G. In similar manner we fill in the cells of the third and fourth rows of Table G.
8. The last step is to add the elements in each column of Table G to obtain the sums representing the "total" relative importance (the indirect normalized weight) of each of the six joint-actions for achieving the LDCs' overall goal. If this non-mathematical approach is the one being adopted, then the six figures would be entered in the boxes for LDCs in the bottom level of the hierarchy (Diagram 1).

Table G: Computation to Derive the Indirect Normalized Weight for Each Joint-Action

	GATT-TRA	GATT-ETR	DIPL-TRA	DIPL-ETR	PROT-TRA	PROT-ETR
MARKET	0.0055	0.0119	0.0053	0.0229	0.0018	0.0046
GROWTH	0.0692	0.0161	0.0281	0.0077	0.0225	0.0055
EFFICIEN	0.0969	0.0229	0.0486	0.0212	0.0486	0.0068
STABLE	0.0328	0.1232	0.0652	0.2395	0.0333	0.0581
Total	0.2044	0.1741	0.1472	0.2913	0.1062	0.075

9. Repeat the same set of steps for the DCs' hierarchical structure indicated in Diagram 1. It consists of four levels. Thus the "total" relative importance of each of the six joint-actions is determined by: (1) summing their relative importance for attaining each of the four DCs' targets when adjusted for the weights of these targets, where (2) the weights of each of the four DCs' targets are obtained by summing their relative importance for each of the three DCs' objectives when adjusted for the weights of the objectives, where (3) each of the weights of the objective are determined from their relative importance for attaining the DCs' overall goal. Put in another way, the total relative importance (indirect normalized weight) of each of the joint-actions is thus obtained via their direct effects on the targets where the targets' normalized weights are obtained via their direct effects on the objectives, where the objectives' normalized weights are obtained via their direct effect on the overall goals.

The resulting relative importance of each joint-action for attaining the DC's overall goal is listed at the bottom level of the hierarchy.

IV. Extension and General Applicability of AHP

To repeat, the first step of this procedure involves the setting down (determination) of a relative importance scale for use in making pairwise comparisons --- a ratio scale that the analyst finds relevant for the conflict situation, and especially the perspectives and knowledge base of participants in the conflict.

However, in a game-type situation it is not necessary for each participant to use the same ratio scale. In general, the finer (coarser) the grain of the ratio scale that is used, the more (the less) precise the resulting relative preferences. So perhaps a mediator or third person might suggest to each party a scale consistent with the mediator's perception of the party's ability to handle pairwise comparisons. In this way more accurate evaluations of each party's relative preferences may be obtainable.

More important, the resulting relative preferences of each party can at times be extremely helpful to the mediator in suggesting a compromise solution. For example, take the relative preferences of the DCs and LDCs reported in Diagram 1 for the set of six joint actions considered. They are set down in Matrix I, which depicts a non-equilibrium situation. Although the numbers

Matrix I. World Economic & Trading System: Non-equilibrium Case

		Developed Countries (DCs)		
		Quiet Diplomacy (DIPL)	*Support GATT/WTO (GATT)*	*Trade Sanction (PROT)*
Developing Countries (LDCs)	*Ec & Trade Lib, Human & Lab Rights & Env (ETR)*	(.298; .258)	(.176; .211)	(.074; .350)
	Focus on Econ & Trade Liberalization (TRA)	(.143; .071)	(.205; .043)	(.104; .067)

recorded cannot be taken to be objective scientific measures of the absolute desirability of the joint actions, let alone to permit interpersonal comparisons, they can yet be extremely useful to a mediator (or third person). The mediator can easily point out that the DCs have a strong preference for the LDCs to choose action ETR; the payoffs for the DCs for its three possible actions are

	DIPL	GATT	PROT
ETR	.258	.211	.350
TRA	.071	.043	.067

Clearly, the choice of ETR by LDCs rather than TRA is much preferred by the DCs for each action they consider. It would seem that by pointing to these differences in this table which record as precisely as the DCs can state their relative preferences, the mediator may be able to persuade the DCs to choose the action DIPL rather than experience a succession of actions and reactions that would be involved in a non-equilibrium situation, actions and reactions that could yield them the lower outcomes in the lower part of the columns of Matrix I. Recall from Matrix I that the LDCs would select ETR and not deviate from it since the joint action DIPL/ETR is the one most preferred by the LDCs.

This ability to persuade the DCs to commit themselves over both the short-run and long-run to choose DIPL would be enhanced if in fact the relative preferences of the DC's were say

	DIPL	GATT	PROT
ETR	.308	.161	.350
TRA	.071	.043	.067

where the payoff in the top of the first column is taken to be 0.050 greater and that of the second column, 0.050 smaller.

One can proceed to speculate even further about the reactions of the DCs with regard to the relative preferences that might be revealed, and the mediator's ability to set forth a persuasive argument. This direction of research obviously needs much further exploration. In this article, we simply wish to demonstrate that the deviation of rather precise relative preferences for each party in a conflict can often provide useful information for reaching a compromise position without the use of interpersonal comparisons.

V. Mathematical Statement of the Steps

To a mathematician, the normalized weights described in the preceding section for each party are only approximations. Given any pair-wise matrix such as those in Tables B, D, and E, the more exact ranking could be obtained by normalizing the eigenvector of each matrix. In particular, the relevant eigenvector would be the one based on the maximum eigenvalue of the matrix. The following describes the derivation of such eigenvector and eigenvalue.

Let A_1, A_2, A_3, …, A_n be ***n*** elements in a level. The quantified judgments on pairs of elements (A_i, A_j) are represented by an ***n-by-n*** matrix **A** = (a_{ij}); i,j = 1, 2, 3,....., n. A set of numerical weights w_1, w_2, w_3, ..., w_n reflects the recorded quantified judgments. Hence, in paired comparisons:

$$\mathbf{A} = \begin{array}{c} \\ A_1 \\ \\ A_n \end{array} \overset{\begin{array}{ccccc} A_1 & A_2 & & & A_n \end{array}}{\begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots\cdots & \cdots & w_1/w_n \\ .. & .. & .. & & .. \;\; . \\ w_n/w_1 & w_n/w_2 & \cdots\cdots & \cdots\cdots & w_n/w_n \end{bmatrix}}$$

Elements A_1, A_2, ... A_n are compared pairwisely. Since every row is a constant multiple of the first row, **A** has unit rank.

By multiplying **A** with the vector of weights ***w***, one will have

$$\mathbf{A}\boldsymbol{w} = \boldsymbol{nw} \qquad (1)$$

To recover the scale from the matrix of ratios, the following system of equations must be solved:

$$(\mathbf{A}-n\mathbf{I})\boldsymbol{w} = \boldsymbol{0}$$

Clearly, a nontrivial solution can be obtained if and only if **det(A-*n*I)**, which is the *characteristic equation* of **A,** vanishes. Hence, ***n*** is an *eigenvalue* of **A** and ***w*** is the corresponding *eigenvector.* Because **A** has unit rank, all its eigenvalues except one are zero, and the only non-zero eigenvalue is consequently a maximum.

If each entry in **A** is denoted by a_{ij}, then $a_{ij} = 1/a_{ji}$ (reciprocal property) holds, and so does $a_{jk} = a_{IK} / a_{ij}$ (consistency property). By definition, $a_{ii} = a_{jj}$ = 1. Therefore, if we are to rank ***n*** elements, and thus **A** is ***n***-by-***n***, the required number of inputs (from the paired comparison) is equal to **$(n^2$-n)/2,** since the reciprocals are forced. Thus, six judgments are needed to compare four elements (targets) of LDCs in Diagram 1; those are reflected in the cells of one of the off-diagonal sections in Table D.

In general case, the precise value of w_i/w_j is not given, simply because the input judgment is only an estimate of w_i/w_j. The a_{ij} may be regarded as perturbations of w_i/w_j. While the reciprocal property still holds, consistency does not. If we denote the largest eigenvalue by $ë_{max}$**,** then, by perturbation theorem, (1) becomes:

$$\mathbf{A}\, \boldsymbol{w} = \text{ë}_{max} \cdot \boldsymbol{w} \qquad \textbf{(2)}$$

where **A** is the actual, or the given matrix perturbed from the matrix $\mathbf{w_i/w_j}$. Despite the difference between (1) and (2), if $\boldsymbol{w}$ is obtained by solving (2), the matrix whose entries are w_i/w_j is still a consistent matrix; it is a consistent estimate of **A,** although **A** itself needs not be consistent. Notice that **A** will be consistent if and only if $\text{ë}_{max} = \boldsymbol{n}$. As long as the precise value of w_i/w_j cannot be given, which is common in the real case due to bias in the judgments, ë_{max} is always greater than, or equal to $\boldsymbol{n}$. Hence, a measure of consistency can be derived based on the deviation of ë_{max} from $\boldsymbol{n}$ (the conditions for existence of an eigenvalue under a small perturbation and for the stability of an eigenvector are shown in Saaty, 1994). Returning to the LDCs' hierarchy, as an example we will now show the steps for obtaining the ranking at the bottom level of the hierarchy, i.e., the joint-actions.

Each row of the 6-by-4 matrix in Table H is the eigen-vector of the pairwise matrix that constitutes the relative importance of the six joint-actions with respect to each of the LDCs' four targets. The ranking of the targets itself is shown by vector EIG-0 in the middle of Table H, with the following numbers: 0.052, 0.140, 0.250, 0.558. Notice that the numbers produced by a non-mathematical approach described earlier (on the left column of Table F) are very close to these numbers. Finally, to obtain the ranking of the joint-actions, one has to multiply the 6-by-4 matrix with the vector EIG-0; this yields a vector denoted as FINAL in Table H. Again, notice that the elements of this vector are very close to those obtained by a non-mathematical approach (the bottom row of Table G). The numbers recorded for LDCs at the bottom level of the hierarchy in Diagram 1 are precisely those taken from the vector FINAL.

Table H: Final Ranking of LDCs' Strategies Through Eigen Vector and Vector Multiplication

	EV-MARKET	EV-GROWTH	EV-EFFICIEN	EV-STABLE		EIG-0		FINAL
GATT-TR	0.099	0.486	0.398	0.057		0.052		0.205
GATT-ET	0.232	0.097	0.093	0.228	X	0.14	=	0.176
DIPL-TRA	0.096	0.189	0.197	0.112		0.25		0.143
DIPL-ETR	0.455	0.047	0.086	0.441		0.558		0.298
PROT-TR	0.033	0.146	0.197	0.059				0.104
PROT-ET	0.085	0.035	0.028	0.103				0.074

VI On the Consistency Issues

The AHP uses a *consistency index* (CI), which is equal to $(\text{ë}_{max} - \boldsymbol{n})/(\boldsymbol{n} - 1)$. Comparing CI with the average *random index* (RI), which is the consistency index calculated from a large sample of generated reciprocal matrices, one can form a *consistency ratio* (CR), which is the ratio of CI to the average RI. This ratio can alternatively be stated as the *overall inconsistency index* (OI). The threshold point is usually OI $\leq$ 0.10, which indicates a one-level or lower order of magnitude adjustment in the judgments. The size of OI for the conflict situation that we are dealing with, is shown in Diagram 2 of Azis' paper (Azis, 1996), i.e., 0.07 and 0.08 for LDCs and DCs respectively.

When more than two elements are compared, the notion of consistency can be associated with the assumption of *transitivity*: if $A_1 > A_2$ and $A_2 > A_3$, then $A_1 > A_3$. It should be clear, that in solving for ***w*,** the transitivity assumption is not strictly required, because inconsistency may arrive from the lack of precise relations among the judgments even if they are transitive. Because the AHP allows for inconsistency, the judgments do not have to be fully consistent (in fact, in addition to permitting some degree of inconsistency, another strong point of AHP is that it allows for rank reversal to occur when it is desirable for that to happen (Saaty, 1994). Yet, as shown earlier, the resulting matrix and the corresponding vector remain consistent. It is the consistent vector ***w*** that reflects the priority ranking of the elements in each level of the hierarchy.

VII Concluding Remarks

We have described two approaches in determining the relative importance of a set of meaningful joint-actions in a particular conflict situation where the two parties have different value systems, where the parties may use different scales, and where in particular inter-cardinal comparisons are difficult to make. One is where participants wish to be able to replicate calculations that might be made by a mediator, arbitrator or other interested third party. For this situation, the non-mathematical approach is appropriate when each participant does not have access to mathematical knowledge. The second approach, a mathematical one, is more precise and formal, which may appeal for its more advanced and sophisticated analysis.

The weights recorded in Diagram 1 were derived with the latter approach (column FINAL in Table H). Using the example of LDCs' hierarchy, it is shown in the last row of Table G that the non-mathematical approach is capable of providing very similar (very close) ranking values.

Finally, while interpersonal comparisons cannot be made, a mediator or third party may nonetheless find the non-comparable relative values of the participants extremely useful in certain cases to suggest an acceptable compromise otherwise not obtainable.

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