The use of mathematics in social sciences, including economics, has increased dramatically since the 1930s. Many areas used to be analyzed in a literary sort of way have become amenable to mathematical modelling. The progress in the development of computer hardware and software has allowed social scientists to explore more complex systems. Social sciences and economics themselves also evolve. They require an increase in precision with which researchers could communicate the facts that they have discovered. In this sense, mathematics is like a language, a means of communication, using which the approach of mathematical modeling tries to explain a complex socio-economic system. Although useful—in some cases also inevitable—linking mathematical models with economic concepts and theories is often challenging. For many, it is a strenuous task. This book is intended to help facing the challenge. By using selected issues and examples, the main purpose is to show how mathematical models are used for socio-economic analysis. Readers and students interested in understanding and building mathematical models for such analysis will benefit from reading this book.

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Selected Topics and Applications

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PREFACE

More-than two decades of teaching and doing research at Cornell University have reinforced my conviction that students who may be doing well in math and economic theory courses could still encounter difficulty in building relevant mathematical models for economic analysis. The evidence is clearest when they are about to pick topics for their theses and dissertations. Selecting a topic with good mathematical models is always strenuous for them. The reasons behind this are various, but I am convinced that a lack of understanding about what the actual problems and the issues to be researched are, is one of them. The difficulty to link those problems with the economic theories they learned during their years at college is next on the list. At any rate, the problem has little to do with their knowledge and ability to do math or economic theory.

Almost every year before the semester began I felt a yearning for writing a book to help students overcome the predicament. The subject I had in mind was on applications of mathematical models for the analysis of actual and specific economic problems. However, a combination of my limited time, failure to meet some deadlines, and too numerous topics to cover have prevented me from doing it. Even at the time of writing I still have two books to finish, the contract on which I have already signed with the publisher back in 2011. Unfortunately, the topic of the two has nothing to do with mathematical modeling.

My subsequent appointment with an international organization made it even more difficult to spend time to write the book. Although I continue to believe that such a topic is so important for helping students to come up with quality theses, my schedule and responsibility along with the constant travels I was required to make had caused my plan to write that book a thing of the past.

Upon finishing my appointment with that organization, I resumed my teaching at Cornell but this time I decided to do it only for one semester each year and spend the other semester abroad, including at the Faculty of Economics and Business, University of Indonesia (FEB-UI), my old alma mater. Helping FEB-UI with their curriculum and teaching some courses, including online courses, have been my main tasks. The courses I was asked to teach were those they believe need to be improved and updated. Most of them are basic and fundamental courses like microeconomics, macroeconomics, and economic modeling; others are on more specific subjects. Since I have taught most of those courses at Cornell, there was not much preparation I had to make. It is unclear if that was the triggering factor, but precisely during that time my early thought of writing a book on mathematical modeling came alive again. After discussing with some colleagues and considering the demand and suggestions from students and the TAs, I decided to write the book. And here it is.

My first and foremost debt of gratitude is to all my present and former students across the globe. It was through discussions and exchanges with them that I could develop a better understanding about what students need and what approach I should take to cater that need. The time I spent to prepare the book could have not been productive without the constant support from Erina, who stood beside me with "Heal" throughout writing this book, and who always pushes me to share knowledge with others. In rushing to prepare the manuscript, I received help from Fandy, Nabil, and Canyon whom I asked to read and edit the draft at the speed of a *Shinkansen*. I am grateful for their help. But any short-comings of the book are my responsibility.

CHAPTER 1

INTRODUCTION

A mathematical model for economic analysis is described as a formal description of relationships between variables. Some of those relationships are derived from empirical observation, others are deduced from theoretical axioms based on assumed behavior of the economic agents.

In microeconomics, examples of the relevant variables are quantities and prices of outputs and inputs, transactions costs associated with the location of producers and consumers, and the market structure. In macroeconomics, the applicable variables could be the size of aggregate output and prices, employment, the level of saving and investment, etc. In finance, things like the volume of credit, the size of equity market and bond markets, the variety of other types of securities, exchange rates, and cross-border flows of capital, are examples for which economic models are often used. How those variables interact, and what the implications of such interactions would be are questions which mathematical models in economics could help shed some light on.

But economics is a social science, so why mathematical models? Is it because mathematics itself is a science? It has been argued that mathematical models, like mathematics, are both science and art. Mathematical models may have been developed based on observations and theories, and hence have an element of science. Yet, the process of building mathematical models are also intensely creative. Making theorems or abstracting a complex economic system from nothing is much like the way a writer writes a novel or a music composer composes a song from a blank paper. In both, patterns are to be detected or invented, for which the modeler's expressiveness is limited only by his/her imagination. The challenge with thinking of mathematical models as an art is that it is harder to convince an economist to read mathematical models or sit through a lecture on economic modeling than to lure them to enjoy music, a good novel, or a well-crafted painting.

As an analyst, we observe the world and attempt to develop theories and models based on those observations. Subsequently, we validate those theories and models by using existing data before using them to predict the future, or to design policies. Hence, economic models play an important role not only in the description stage but, more importantly, also in the prediction phase. The decision and quality of policy are likely enhanced by the accuracy of model-based prediction. Indeed, the ability to describe and predict is one of the important features of economic theories and models. Without the power of prediction, those theories and models are history at best, and story at worst, with no meaningful and logical implications. Of course, data and basic hypotheses of the model will influence the final results: as the maxim goes, "garbage in, garbage out."

This book is about mathematical models relevant for economists and decision makers. From the book's perspective, mathematics or mathematical models are like a language, a method or a system of communication, the bridge between art and science. They are used to describe mundane facts like economic booms and busts, profit and loss, prosperity and poverty, and they can also be used to invent patterns and rhythms like those embedded in beautiful music, painting, or poetry.

While models have their formal, scientific and epistemological definitions, the book's emphasis is on the applications of those models. A book on mathematical economics would usually begin with the basic concepts of mathematics relevant to economic analysis. They typically include logics, sets, real numbers, discrete and continuous functions, convexity, vectors, differentiation, integration, and matrices. Although those concepts are critical for understanding economic models (and can be found in numerous textbooks or other forms of publication), they are not what I intend to cover here. Instead, the purpose of this book is to show the applications of mathematical approaches and models in economic analysis using some examples of micro and macroeconomic issues. It is intended as a reading material to accompany courses and lectures in economic modeling, mathematical economics, and/or other topics of that nature. The emphasis is to combine the concepts and models with their applications, and the goal is to help readers better understand how those models are used–and be able to conduct modeling work–for economic analysis.

For that purpose, several examples of model applications using data and computer software are presented, most of which are based on my previous work, published and unpublished. Aside from those examples, I also list some exercises (problem sets) for the differential equation in a dynamic system discussed in Chapter III, the computable general equilibrium (CGE) and financial computable general equilibrium (FCGE) model in Chapter VIII, and for the game theory in Chapter X.

One of the most difficult tasks in preparing the book manuscript is to select examples for each model. Coming up with comprehensive examples which match with the design of the models is never easy. Being comprehensive could mean covering economic, social and environmental subjects ('sustainable development' concept). The microeconomic examples are shown in the non-linear programming topic in Chapter II (price differentiation), and in game theory, the AHP, and the ANP in Chapters X, XII, and XII, respectively. Meanwhile, macroeconomic examples are highlighted in most Chapters. The social issues related to regional disparity or decentralization are taken up in Chapters II, III, XI, and XII. Income inequality and poverty are discussed in Chapters VII and VIII, and the subject of endogenous institutions in decentralization is taken up in Chapter X. Examples with environmental dimension are used to analyze the link between economic efficiency and pollution reduction by using a CGE model in Chapter VI, and the economy-wide impact of climate change using an FCGE model in Chapter IX.

Being comprehensive in examples could also mean covering regional (sub-national), national, and global dimension. Some discussions throughout the book reflect such a coverage. Yet, comprehensiveness could also be judged based on the extent to which the real sector (domestic and foreign) and the financial sector are integrated. With such an integration, the economy-wide impact of a major event, say, a financial crisis, can be well captured and analyzed in the model system. The discussions on FCGE in Chapters VIII and IX demonstrate this scenario, the data system for which is elaborated in Chapters IV and V (the social accounting matrix SAM and the FSAM). Meanwhile, the specifications of the core equations are discussed in Chapter VII.

To deepen readers' understanding about the advantage of using models for economic analysis, I show some counterfactual simulations using the CGE and FCGE models in Chapters VI, VII and VIII. While, in general, examples and counterexamples are extremely important for economic analysis, they are particularly useful for evaluating/selecting alternative policy measures. They can enhance the quality of policy debate and help reveal a deeper theory.

The classification of models into 'forecasting' and 'planning,' elaborated in Chapter IV, is also useful in understanding the distinction between the two, and to capture their interaction in order to enable the model to provide a more complete analysis. Although 'planning' models seem more relevant for decision making because the policy variables are endogenously determined, it is 'forecasting' models that could show what is possible and what is not. And this is very important in order to avoid a wishful thinking scenario. After all, economic policies must be predicated not on an ideal world but on the world as it is.

Another feature common in almost all cases is to compare the benefits, including the opportunities, and the costs including the risks, of any policies or decisions. The use of that feature is shown in the application of the Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP) discussed in Chapters XI and XII, respectively.

It is well-known that the goal of all scholarship is to provide an understanding of the principles that govern the world. For economics, it is no exception. This necessarily means developing models on which one can perform thought experiments to answer question like "How would things be different if X happened instead of Y?" Although most models and theories discussed in the book are useful for policy and decision making, the purpose of mathematical models in economics can go beyond that. They can be used more specifically to analyze and find patterns, be it economic or social patterns, and to verify or falsify specific statements concerning the objects to be studied in micro and macroeconomics. Examples are statements about prices, location, competition and monopoly, household income and purchasing power, gross domestic product, unemployment, poverty and income distribution, interest rates, exchange rate and capital flows.

While models are useful, it is important to note that the results and implications of a model-based analysis depend on the model specifications and assumptions, which may or may not conform with reality. The conclusions derived from model simulations should therefore be taken with caution. After all, models are simplifications of reality; they are to be used but not to be trusted.

With regards to interpreting the results to draw policy implications, mathematical models, including those presented in this book, may give a set of logical solutions, but interpreting those solutions and deriving policy implications require aptitudes and senses related to motivation, design, and empathy. The nature and intensity of these aptitudes and senses could differ between users. It is in this perspective the discussions about the findings throughout the book should be interpreted.

To the extent reading and understanding models are different from actually building and using them, and given the goal of the book is to help readers to use the mathematical concepts and models for economic analysis, it is imperative for readers to conduct the analysis by developing and using some of those models. “You can’t cross the sea merely by standing and staring at the water” (Rabindranath Tagore).